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Comparison of electron bunch asymmetry as measured by energy analysis and coherent transition radiation

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The longitudinal electron density in millimeter long relativistic electron bunches is determined from the coherent transition radiation spectrum using a Kramers-Kronig minimal phase analysis technique. The bunch shape is also measured by superimposing a position-dependent energy slew on the bunch and then energy analyzing the beam. The results show that the minimal phase analysis correctly describes the asymmetric electron density distribution. [S1063-651X(97)50210-8]

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Ambitious future plans for short charged particle bunches make use of linear colliders with bunch lengths (rms) at the interaction point specified to be from 500 μm to as low as 90 μm [1,2]; consequently, techniques to monitor such short bunches are attracting increasing interest. Classical monitoring techniques employ streak cameras [3]. Several other methods, including deflecting rf cavities [3] or phase sensitive detection with bunch monitors [4] have been investigated; however, all reach their limits when structures much less than 3 ps ($\sim$1 mm) are to be identified. A developing technique is the measurement of coherent synchrotron radiation (SR) or transition radiation (TR), which is emitted at wavelengths comparable to or longer than the charged particle bunch [5–10]. In 1994 it was proposed [11] that coherent SR or TR could be used to determine the complete asymmetric longitudinal bunch shape; even though, the phase determination necessary for such an analysis of an asymmetric bunch is, in principle, multivalued [12]. Although a numerical analysis has shown that the minimal phase solution reproduces analytical pulse shapes very well [13], independent experimental evidence of the coherent TR or SR determined bunch shape has been lacking [14].

In this paper coherent TR measurements of the longitudinal shape of short electron bunches are compared with those determined from a position correlated energy slew method [15]. The results are that both techniques give similar longitudinally asymmetric bunch shapes. This comparison experimentally confirms that neglecting the Blaschke phase contribution in the coherent radiation analysis is a valid approach when determining the asymmetric bunch shapes encountered in accelerators [12].

The experiments were performed at the injector linear accelerator (LINAC) for the Cornell synchrotron. It is a 2856 MHz *S*-band accelerator that consists of a pulsed thermionic triode gun, a two-stage subharmonic prebuncher, and eight accelerating sections. After accelerating through section 5, the beam has an energy $\sim$200 MeV. The bunch shape will not change over the 20 m distance of the last three linear accelerating sections, since the velocity spread of the highly relativistic electrons is very small. To minimize the effects of beam loading or wake fields the LINAC is operated in a single bunch mode only, at a repetition rate of 60 Hz and with a moderate bunch charge of 2×10^9 electrons. At the end of the LINAC is a 0.5 T 18° bending magnet, which is used to inject the electrons into the Cornell synchrotron. Making use of downstream movable collimators, the bending magnet can be used as an energy analyzer of the electrons. When the magnet is switched off, the electrons travel through about 1 m of beampipe and through a flat metal film, where the coherent TR is produced [8].

The coherent TR, emitted at 90° to the electron beam trajectory, is spectrally analyzed with a Michelson interferometer [14]. A millimeter-wave spectrum obtained from such a measurement is represented by the solid line in Fig. 1(a). A reliable spectrum can be measured down to frequencies $\omega=1/\lambda\approx1.5$ cm^{-1}. To obtain the asymmetric bunch shape from the Kramers-Kronig analysis it is necessary to know the form factor to zero frequency. The extrapolation procedure to extend the data on the high and low frequency side has been described in some detail previously [12–14]. The resulting low frequency extrapolation is represented by the dotted line in Fig. 1, while the high frequency extrapolation is obtained by extending the high frequency data with a power law $\sim\omega^{-4}$. It has been argued that the Blaschke phase contribution is negligible for realistic bunch shapes [12,13], so that the minimal phase obtained from the Kramers-Kronig analysis is the correct input with which to determine the asymmetric shape of the electron density distribution.

Figure 1(b) shows the electron bunch shape as calculated with this technique. It consists of a peak with a full width at half maximum (FWHM) of about 1.2 mm and a long tail, which indicates a pronounced asymmetry. One ambiguity of this analysis technique, due to time reversal symmetry, is that either side could represent the leading edge. A more serious potential problem is that if for some reason the Blaschke phase is not negligible then the phase determination is not unique and neither is the bunch shape presented in Fig. 1(b).

The independent measurement of the longitudinal shape of these bunches uses a position correlated energy slew to determine the bunch shape. In a similar fashion the entire phase space of the bunch can be mapped as discussed, for instance, by Crosson *et al.* [16]. We are interested in the projection of phase space which provides the longitudinal electron density distribution. To determine it, we use only

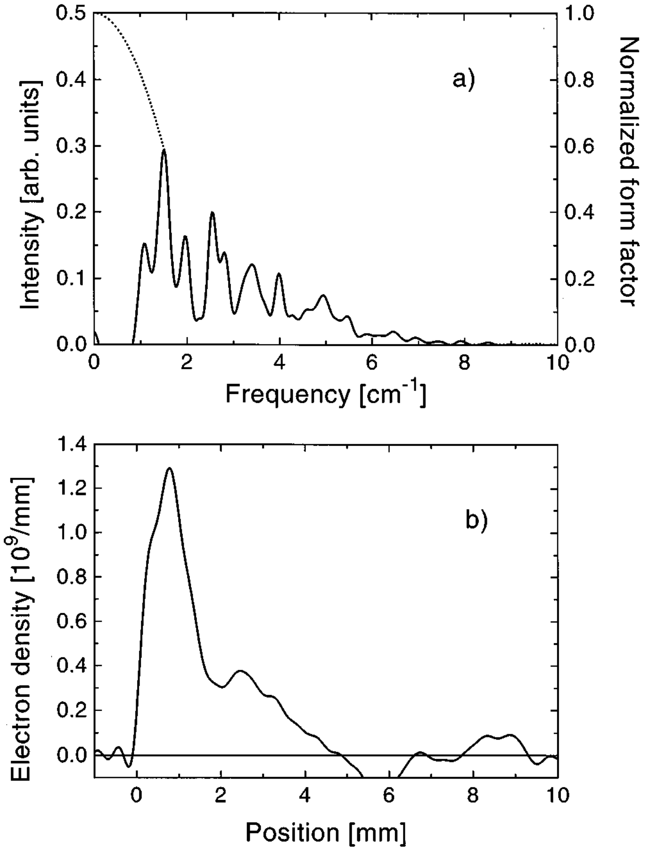

FIG. 1. Measured coherent transition radiation spectrum and calculated longitudinal bunch shape. (a) The intensity spectrum in arbitrary units as a function of frequency (solid line, left ordinate). The dotted line shows the low frequency extrapolation and normalization used for the form factor (right ordinate). (b) Asymmetric bunch shape versus position calculated from the transition radiation spectrum under the minimal phase condition.

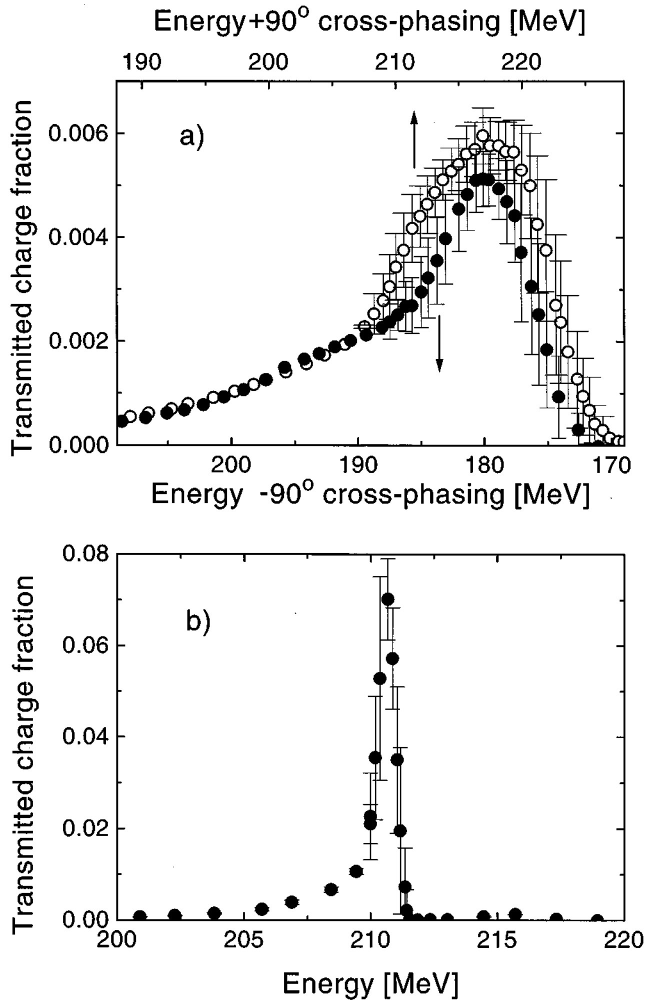

FIG. 2. Electron energy distribution of the bunch. (a) After acceleration in sections 1–5 and after imposing a position-dependent energy slew of about 10 MeV/mm on the bunch by cross-phasing accelerator sections 6–8. Solid circles and bottom scale are for a −90° phase setting, open circles and top scale for +90° cross-phasing. (b) After acceleration to about 200 MeV by the first five accelerating sections and sections 6–8 not pulsing. Plotted in both cases is the ratio of the charge passing the 400 μm aperture 4.5 m downstream of the bending magnet to the bunch charge before the bending magnet versus the electron energy, determined by the field setting of the bending magnet. The error bars indicate the rms fluctuations of the charge measurement.

the first five sections of the LINAC to accelerate the beam and employ the last three accelerator sections to introduce a position correlated energy slew on the electron bunch. This method—though conceptually simple—requires accurate tuning of the LINAC, is limited by its energy jitter and the energy acceptance of the focusing elements, and, furthermore, requires accurate knowledge of the accelerating rf field and wake field effects. Therefore this type of measurement requires dedicated LINAC operation, produces lower spatial resolution results (~100 μm), and is much more time consuming than a SR or TR measurement of the bunch shape.

The procedure is as follows. Downstream accelerator parameters are optimized to obtain stable operation with one set of bunching parameters. Accelerator sections 6–8 have their rf phases shifted from the maximum acceleration setting in the same direction by 90°, so that the electron bunch is located at or near the zero crossing of the electric field (termed ''cross-phased'' below) giving a linear correlation between the position of an electron within the bunch and the energy it acquires from the accelerating electric field in sections 6–8. Beyond section 8 the 18° bending magnet is used as an electron energy analyzer by inserting collimators making a beam pipe aperture of 400 μm at a distance 4.5 m downstream from the magnet. Including the 1 mm transverse electron beam diameter, we expect a theoretical energy resolution of about 100 keV, which is increased by the LINAC energy jitter of a few 100 keV. We measure the bunch charge before the bending magnet and 1 m after the scrapers. The fraction of electrons transmitted as a function of magnetic field is then linearly related to their energy and correlated with their position in the bunch.

The calibration constant that relates the bending magnet's field setting to the particles' energy is determined from measuring the field strength with a Hall probe at different control settings, corresponding to different injection energies of the synchrotron. A second calibration constant ΔE_z, the slope of the energy gain vs longitudinal position, comes from the rf wavelength and the maximum energy gain for sections 6–8, determined by the difference in beam energy with the sections on and off. The last three accelerating sections can

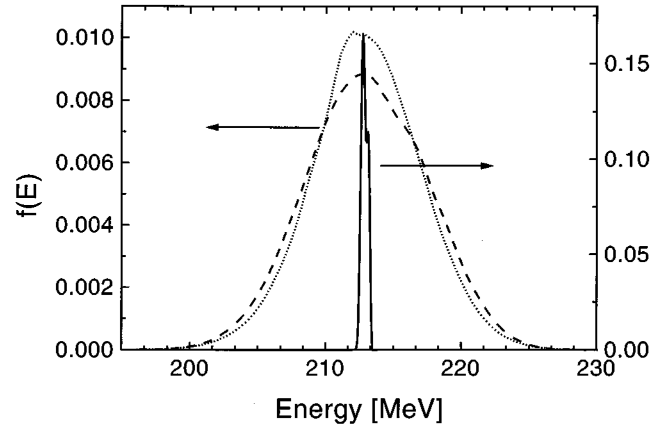

FIG. 3. Calculated energy distribution $f(E)$ for a Gaussian bunch with a FWHM of 1.25 mm. The solid line is for a 1.5° off crest acceleration through sections 1–5, including wake field effects and accelerator sections 6–8 not pulsing. Dotted and dashed lines are for accelerator sections 6–8 cross-phased by +90° (dotted line) and −90° (dashed line) with respect to the first five sections.

provide about 140 MeV of peak energy gain to the electron bunch, so that a longitudinal energy gradient, $\Delta E_z = 8.3$ MeV/mm is superimposed on the bunch. This gradient is determined to a 5% absolute accuracy in this measurement.

Figure 2(a) shows the electron density as a function of energy for a cross-phasing of +90° (open circles and top scale) or −90° (solid circles and bottom scale). The difference in centroid energies is presumably due to the actual phase changes not being exactly ±90°. The ratio of the FWHM of the two distributions is (13.1 MeV)/(10.7 MeV) =1.22. Ideally, both phase settings should give the same electron density distribution, centered at the same energy, only with a reversal of the high and low energy particles. This is clearly not the case in our measurement and is an indication that there already was an energy slew within the bunch before arriving at section 6 caused by (i) the bunch not riding exactly at the crest of the accelerating field of sections 1–5, (ii) the cross-phasing angle of sections 6–8 not being exactly 90°, or (iii) the existence of wake fields correlating the longitudinal position and a particle's energy. To estimate (i) and (iii) a measurement of the electron energy distribution has been performed by removing the triggers for the modulators and, hence, switching off the accelerating fields of sections 6–8. The result is shown in Fig. 2(b). The FWHM is 0.88 MeV, larger than the energy resolution and the expected energy jitter. Thus the effect of the slew of electron energies through the bunch before the cross-phased sections has to be taken into account.

To obtain an estimate of this preexisting energy slew, a numerical calculation is performed to calculate the normalized energy distribution function $f(E)$. A phase shift ϕ between the bunch and the crest of this sinusoidal rf field is an input for the calculation and the effect of the wake fields of the Cornell LINAC is approximated using the results from Bane and Weiland [17] for the Stanford Linear Accelerator (SLAC) structure assuming a Gaussian shaped bunch with 2.5×10^9 electrons with FWHM=1.25 mm. These parameters correspond to the bunch width measured in the TR experiment. With the first five sections accelerating the single bunch, ϕ is varied until $f(E)$ is calculated with a

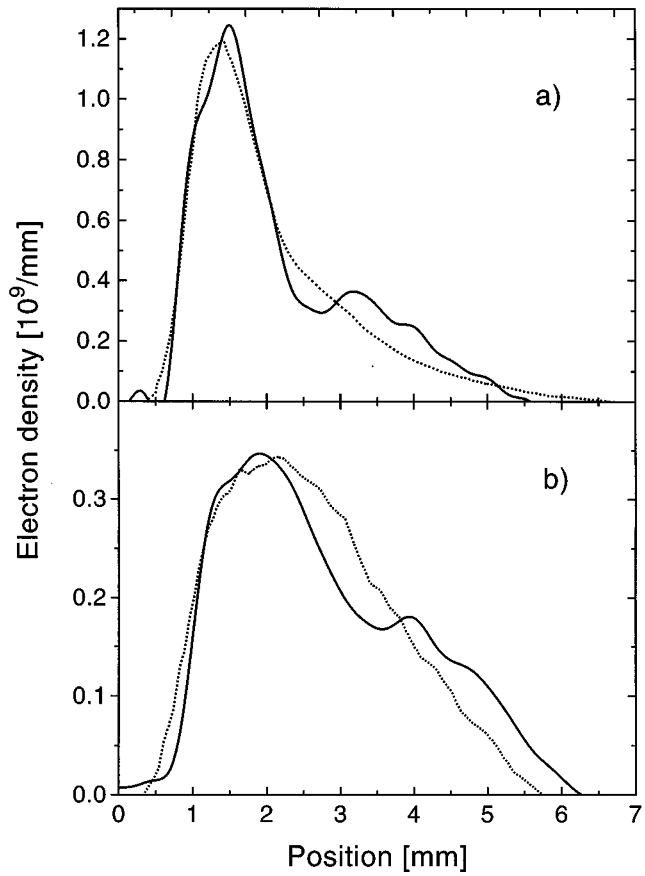

FIG. 4. Comparison of the bunch shapes obtained from the higher resolution TR measurement and the lower resolution energy slew technique. (a) A narrow electron bunch; (b) A wider electron bunch produced by detuning the LINAC prebunchers. Solid lines, analysis of the coherent transition radiation spectrum; dotted lines, results of averaging the bunch shapes obtained from the two cross-phasing settings of the accelerator sections 6–8. Both results were normalized to the total charge of the electron bunch, and the zero levels for the energy slew are determined by the values about 10 mm from the center.

FWHM of 0.68 MeV (after being convolved with the 100 keV energy resolution) at a value for ϕ of 1.5°. This result for $f(E)$ is plotted as the solid line in Fig. 3 and its FWHM makes an appreciable contribution to the observed energy spread, likely accounting for the discrepancy between the +90° and −90° cross-phasing results.

Numerically cross-phasing the rest of the accelerator by ±90° results in the dotted and dashed $f(E)$ shown in Fig. 3. Similar to the experimental results, these two phase settings lead to distributions with different widths. The calculated FWHM ratio of (10.9 MeV)/(9.3 MeV)=1.17 agrees with the experimentally observed value. It is indicative of different energy gradients and, hence, different calibration constants $\Delta E_z^{\pm}$ for the two cross-phasing angles. The important result of this calculation is that the average of these two FWHM values, 10.1 MeV, is approximately equal to the expected energy width of 9.9 MeV FWHM, obtained in the simulation by removing the wake field effects and by phasing sections 1–5 at 0° and cross-phasing sections 6–8 by 90°. These energy gradients may be understood qualitatively for particles near the center of the bunch's charge distribution. The contributions to the energy gradient near the center of the bunch appear in three separate terms: (i) a constant gradient due to not being at the crest of the rf wave, (ii) two gradients of different magnitude due to the cross-phasing not being exactly ±90°, and (iii) the bunch charge times the gradient of the wake field. Averaging the energy gradient for cross-phasing in both directions yields contributions for (i) and (iii) that cancel separately and a contribution for (ii) that is the average cross-phased energy gradient. The different calibration constants $\Delta E_z^{\pm}$ (for ±90°) can be determined relatively for both cross-phasings from some characteristic length of the distribution (e.g., FWHM) since the bunch shape is independent of the cross-phasing angle. Thus the energy widths $E_{\mathrm{FWHM}}^{\pm}$ for each cross-phasing angle, respectively, yield $\Delta E_z^{\pm} = \Delta E_z[1 \pm (E_{\mathrm{FWHM}}^{+} - E_{\mathrm{FWHM}}^{-})/(E_{\mathrm{FWHM}}^{+} + E_{\mathrm{FWHM}}^{-})]$, and the resulting distribution function $S(z)$ is the average of $f_{\pm}(z\Delta E_z^{\pm})/\Delta E_z$, where $f_{\pm}(E)$ are the measured energy distributions for each cross-phasing.

An equivalent numerical check has been performed with a FWHM=3 mm Gaussian bunch, corresponding to the width of the second experimentally investigated bunch shape. The effect of the wake fields is much smaller in this latter case due to the smaller charge density, and the ratio of the calculated widths of the energy distributions for the two phase settings also matches the measured ratios of the distributions' FWHM.

The bunch shape obtained from the energy slew measurement, averaged over both phases and normalized to the bunch charge, is represented by the dotted curve in Fig. 4(a), together with the corresponding result from the TR measurement (solid curve). Additionally, as the phase and the magnetic field are known, it is possible with the help of the energy analysis method to identify the steeply rising edge as the trailing edge of the bunch. Very good agreement between the two pulse shapes and their FWHM is found. The results from the coherent TR show finer structure due to the higher resolution possible with this technique.

With the prebuncher detuned, we performed a second experiment with a wider bunch. Using the same procedures described above, the bunch shape has been measured by the energy slew technique for both phases and averaged to give the result shown in Fig. 4(b). Again the TR bunch measurement analysis is shown for comparison. Although the low frequency TR detector cutoff decreases the precision of the coherent radiation method for bunches longer than 1–2 mm there is still good agreement between the two experimental methods.

We have shown by direct comparison that the energy slew technique and the Kramers-Kronig analysis of the TR spectrum (to obtain the minimal phase information) yield the same asymmetry for short relativistic electron bunches. In addition, the coherent radiation technique gives good results, even when the frequency spectrum is not known over the complete range.

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